2. Envy-free is fair enough

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ABSTRACT

Purpose: The aim of this chapter is to review the role of envy-freeness (EF) as a core principle of fairness in the allocation of both divisible and indivisible goods, and to evaluate its theoretical robustness, practical relevance, and applicability to real-world allocation problems.

Design/methodology/approach: The chapter builds on interdisciplinary literature from economics, computer science, and behavioural studies. It discusses EF and its relaxations (EF1, EFX), alongside other fairness criteria such as Pareto optimality, proportionality, and equitability. The discussion is grounded in theoretical models and supplemented with practical applications including project assignments and school redistricting.

Findings: Envy-freeness emerges as a psychologically plausible and socially robust fairness criterion. While strict EF is often unattainable in real-life scenarios involving indivisible items, its relaxations (EF1, EFX) offer operationally feasible alternatives that still ensure high levels of perceived fairness. Simple algorithms such as round-robin and envy-cycle elimination provide transparent procedures for computing such allocations. Applications in education and organisational settings demonstrate EF's contribution to social welfare, trust, and procedural legitimacy.

Originality and value: The chapter contributes to the ongoing debate on fairness in resource allocation by showing that EF and its relaxations balance efficiency with individual satisfaction. By incorporating insights from behavioural economics and algorithmic design, it advocates for fair division mechanisms that are both rigorous and practical, thus enhancing the stability and acceptability of allocation outcomes in complex social systems.

Keywords: fair division, envy-freeness, pareto optimality, algorithmic fairness, social welfare.

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Introduction

Fair division—whether of goods, resources, costs, or chores—is a constant challenge in society. What is considered fair depends on context and is often shaped by what is feasible or acceptable rather than what might be theoretically optimal (see Figure 2.1).

In this work, we focus on envy-freeness (EF) as a normative notion of fairness in the allocation of goods and resources. The concept originates with Foley (1967), who formulated it in the setting of an exchange economy with divisible goods. Let $N = \{1, \ldots, n\}$ be the set of individuals and A_i the bundle assigned to agent i. Each individual has preferences represented by a utility function $u_i(\cdot)$. An allocation $(A_1, A_2, \ldots, A_n)$ is envy-free if

$$u_i(A_i) \geq u_i(A_j) \text{ for all } i, j \in N$$

Although originally defined for exchange economies, this condition—that no one prefers another's bundle—has since become a fairness criterion in modern fair division models.

Beyond the outcome itself, fair division research also considers the mechanism by which goods are allocated and the trustworthiness of the entity overseeing the process (Burns et al., 2014).

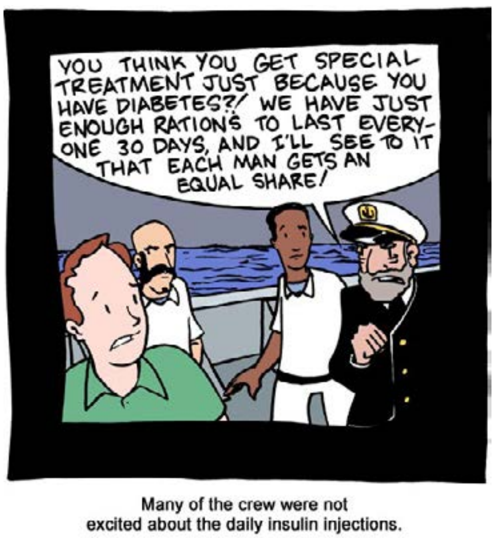

Figure 2.1. Comic from SMBC

Source: Weinersmith (2009).

When EF proves too demanding, especially for indivisible goods, relaxed versions such as envy-freeness up to one good (EF1) or up to any good (EFX) are used. These relaxations preserve fairness intuitions while remaining achievable in discrete settings.

Fair division frameworks are relevant to numerous real-world problems—including school redistricting, food distribution, project assignment, and cost sharing in networks. A key practical challenge is eliciting valuation functions that accurately represent agents' utilities, an issue already noted by Varian (1974).

To evaluate allocations, EF is often examined alongside complementary criteria such as efficiency (Pareto optimality), proportionality, and equitability (Brams et al., 2013; Kurokawa et al., 2018). These principles are not purely theoretical: they inform applications ranging from inheritance division to rent-splitting (Goldman & Procaccia, 2015) and connect deeply with broader traditions in social choice theory, welfare economics, and market design (Moulin, 2003).

2.1. Envy-freeness in fair division: Theoretical foundations

The fair division of divisible goods has been extensively explored using metaphor of "cake-cutting", where a single, perfectly divisible resource—the "cake"—must be shared among agents with subjective preferences. The pioneering work by Steinhaus (1948) revealed both the elegance and complexity of this problem, leading to the development of classic procedures such as Divide-and-Choose, which guarantees an envy-free allocation between two agents and remains a foundational example in fairness education.

Subsequent contributions, including the Moving-Knife and Last-Diminisher protocols proposed by Dubins and Spanier (1961) and later refined by Stromquist (1980), extended envy-free guarantees to settings with multiple participants. By the late twentieth century, comprehensive theoretical frameworks emerged, incorporating additional fairness notions such as proportionality and equitability. Works by Robertson and Webb (1998) and Brams and Taylor (1996) provided systematic treatments of these procedures, demonstrating both the existence of and constructive methods for achieving envy-free or proportional divisions under minimal assumptions.

While some of these protocols are computationally intensive, advances in algorithmic game theory and theoretical computer science have addressed efficiency and strategy-proofness, making fair division increasingly applicable to real-world contexts. Cake-cutting models thus serve not only as elegant theoretical constructs but also as a point of contrast between continuous and discrete divisions—a distinction that remains central in modern fair division research. Over time, it became evident that frameworks designed for divisible goods were insufficient for many practical applications, such as assigning dorm rooms, distributing discrete tasks, or allocating indivisible items that lack natural "cut points".

The study of indivisible goods within fair division gained momentum in the late twentieth and early twenty-first centuries, driven by contributions from game theorists, economists, and computer scientists. Researchers soon recognised that, although envy-freeness is an appealing fairness criterion, it often cannot be perfectly satisfied when goods are indivisible. Real-world examples—such as allocating houses, artworks, or unique assignments—demonstrate this limitation. Consequently, relaxed notions of fairness were introduced, most notably envy-freeness up to one good (EF1) and envy-freeness up to any good (EFX) (Budish, 2011; Caragiannis et al., 2019).

Alongside these theoretical refinements, numerous algorithmic approaches have been developed to compute allocations that approximate or achieve these relaxed criteria under specific conditions. Among the most influential are the Round Robin procedure (RR) and the Envy Cycle Elimination (ECE) algorithm—both simple yet powerful methods that will be described later in this work.

2.2. Common notions of fairness in allocation of goods

For clarity of exposition, this work adopts the discrete formulation of the fair division problem, where the goods under consideration are indivisible. The general framework can, of course, be extended to divisible resources—such as in the classical cake-cutting setting—by interpreting the set of goods as a measurable space and the agents' valuations as additive measures over that space. However, the discrete model provides a natural and practical foundation for the subsequent analysis. Consider a finite set of agents

$$N = \{1, 2, \ldots, n\}$$

and a finite set of goods

$$M = \{g_1, g_2, \ldots, g_m\}$$

A (discrete) allocation is an n-partition of the goods, denoted

$$A = (A_1, A_2, \ldots, A_n)$$

where each $A_i \subseteq M$ is the bundle assigned to agent $i \in N$, and the bundles are disjoint with $\bigcup_{i \in N} A_i = M$.

Each agent $i \in N$ has preferences over bundles of goods, represented by a valuation function

$$v_i : 2^M \to \mathrm{R}_{\geq 0}$$

which assigns a real, nonnegative value to every subset of goods $S \subseteq M$. The value $v_i(S)$ represents the total utility agent i derives from bundle S (Kreps, 1990).

In many fair division models, valuations are often assumed to be additive, i.e.,

$$v_i(S) = \sum_{g \in S} v_i(\{g\})$$

More general (non-additive or submodular) forms can also be considered depending on the application.

Within this setting, several key normative criteria guide what constitutes a fair or acceptable allocation.

1. Efficiency (Pareto optimality, PO)

An allocation $A = (A_1, \ldots, A_n)$ is Pareto optimal if no other allocation $A' = (A'_1, \ldots, A'_n)$ satisfies

$$v_i\left(A_i^{'}\right) \geq v_i\left(A_i\right) \text{ for all } i \in N$$

with strict inequality for at least one agent. In other words, no one can be made better off without making someone else worse off. Although desirable from an efficiency standpoint, Pareto optimality alone does not guarantee fairness—giving all goods to a single agent is trivially PO but clearly unfair.

2. Envy-freeness (EF)

An allocation is envy-free if no agent prefers another's bundle to their own:

$$v_i\left(A_i\right) \geq v_i\left(A_j\right) \text{ for all } i, j \in N$$

This condition ensures perceptual fairness: no agent feels disadvantaged relative to others. Envy-freeness is often associated with social stability, as it minimises discontent or resentment over resource distribution (Foley, 1967).

3. Proportionality (PROP)
An allocation is proportional if every agent receives at least their fair share of the total value of all goods:

$$v_i(A_i) \geq \frac{1}{n} v_i\left(M\right) \text{ for all } i \in N$$

This criterion guarantees that each participant perceives their bundle as worth at least $1/n$ of the total, aligning with intuitive notions of equitable division.

4. Equitability (EQ)
An allocation is equitable if all agents derive the same utility from their assigned bundles:

$$v_i(A_i) = v_j\left(A_j\right) \text{ for all } i, j \in N$$

Here, not only does each agent consider their share satisfactory, but all perceive the outcome as equally satisfying.

In many real-world scenarios involving indivisible goods, achieving strict envy-freeness—or even proportionality—may be impossible. For such cases, relaxed fairness notions have been proposed that preserve the spirit of EF while allowing for discrete constraints.

5. Envy-freeness up to one good (EF1)
An allocation is EF1 if envy that arises can be eliminated by removing at most one good from the envied agent's bundle:

$$\forall i, j \in N, \exists g \in A_j \text{ such that } v_i\left(A_i\right) \geq v_i\left(A_j \setminus \{g\}\right)$$

This relaxation, introduced by Budish (2011), ensures that no agent perceives another's share as substantially better, once a single most-envied item is disregarded.

6. Envy-freeness up to any good (EFX)
An allocation is EFX if the same condition holds for every good in the envied bundle:

$$\forall i, j \in N, \forall g \in A_j \text{ we have } v_i\left(A_i\right) \geq v_i\left(A_j \setminus \{g\}\right)$$

This stronger condition (Caragiannis et al., 2019) is a demanding form of approximate envy-freeness, ensuring that envy disappears regardless of which single good is removed.

These relaxations maintain the intuition of envy-freeness—each agent perceives their allocation as adequate compared to others'—while remaining attainable in discrete allocation problems where strict EF is infeasible.

To better understand why relaxed fairness criteria such as EF1 and EFX are important, consider the following hypothetical examples. Each highlights how fairness notions operate under different assumptions about preferences and indivisibility of goods.

7. Classic birthday party dilemma

Three children—Ania, Bartek, and Celina—must divide five distinct toys: a robot, a board game, a puzzle, a set of crayons, and a soft toy dragon. Their valuations are as follows:

Toy	Ania	Bartek	Celina
Robot	10	8	10
Board game	9	7	6
Puzzle	4	5	5
Crayons	3	4	4
Dragon	2	3	9

Allocation:

- Ania receives the robot and the board game;
- Bartek receives the puzzle and the crayons;
- Celina receives only the soft toy dragon.

Because both Bartek and Celina assign greater total value to Ania's bundle than to their own, the allocation is not envy-free. If one item—the robot—were removed from Ania's bundle, Bartek's envy would disappear, satisfying EF1. However, removing the board game instead does not eliminate envy for either Bartek or Celina, so the stronger condition EFX is not satisfied. This example shows how EF1 can hold even when EF and EFX do not.

8. Two roommates and household chores

Consider two individuals, Mateusz and Kuba, who divide household responsibilities. Mateusz undertakes cleaning the bathroom and washing the dishes, while Kuba is assigned only vacuuming. Mateusz perceives his workload as more burdensome and therefore envies Kuba, indicating that the allocation is not EF. However, if any one of Mateusz's assigned tasks (i.e., either bathroom cleaning or dishwashing) were hypothetically removed from his bundle, his envy would be eliminated. This scenario exemplifies envy-freeness up to any good (EFX)—a relaxation of EF whereby envy is eliminated upon the removal of any single good from the envied bundle. The division satisfies EFX, although it does not achieve full envy-freeness.

9. Proportionality vs. EFX

Consider two student volunteers—Ali and Bea—who are to divide three appreciation gifts for helping organise a university open day: a tablet, a durable backpack, and a premium water bottle. Each volunteer has different subjective valuations:

Item	Ali	Bea
Tablet	55	70
Backpack	10	10
Water bottle	5	10

Total valuations:
Ali = 70 → proportional share = 35
Bea = 90 → proportional share = 45

A proportional allocation is not possible in this case. For example, suppose Bea receives the tablet while Ali gets the backpack and water bottle.

- From Ali's perspective, he receives items worth 15 (10 + 5), which falls short of his proportional share of 35. Thus, the division fails the proportionality criterion for Ali.
- From Bea's perspective, she receives the tablet valued at 70—exceeding her proportional share of 45—so she is more than satisfied.

Now consider the EFX criterion. Ali might envy Bea's single high-value item. However, if that one item—the tablet—is hypothetically removed from Bea's bundle, she is left with nothing, and Ali's envy disappears. Bea does not envy Ali's allocation to begin with. Therefore, while this allocation fails on proportionality (at least for Ali), it satisfies the EFX condition.

These examples demonstrate that envy-freeness up to one good (EF1) and envy-freeness up to any good (EFX) are practical tools for managing the complexities of real-world resource allocation—situations in which goods are indivisible, preferences differ, and perfect fairness is rarely achievable. In applied settings, balancing these relaxed fairness notions with other normative criteria—such as proportionality and Pareto optimality (PO)—often yields the most acceptable compromise between efficiency, individual satisfaction, and collective welfare.

Designing a fair allocation mechanism requires not only a sound mathematical foundation but also a system that all participants can understand, trust, and accept. Transparency—where all agents are aware of one another's allocations—contributes to the perceived legitimacy of the outcome. In practice, however, constructing a procedure that is simultaneously efficient, fair, and transparent is far from trivial (Brams et al., 2013). Hence, the importance of clear, mathematically guaranteed fairness criteria that are both interpretable and reliable. Among these, envy-freeness stands out for its intuitive appeal: it directly incorporates each agent's subjective perspective and, when satisfied, tends to yield allocations that are stable and broadly acceptable, even among agents who do not fully trust one another (Massoud, 2000). For divisible goods, EF allocations are guaranteed to exist; for indivisible goods, relaxed variants such as EF1 and EFX offer practical approximations.

The relationship between fairness and efficiency is not always straightforward. While some envy-free allocations are also Pareto optimal, the two concepts need not coincide. To illustrate this tension, consider a simple scenario involving two individuals with identical shoe sizes.

10. Fairness versus efficiency—The shoe exchange
Two individuals, A and B, wear the same shoe size. Each owns one pair of shoes:

- A's shoes are stylish but uncomfortable;
- B's shoes are plain but comfortable;
- Both prefer comfort to style, and both dislike A's shoes;
- Both prefer a matched pair than a mismatched one.

Initial allocation: each keeps their own pair. This situation is Pareto optimal, since no reallocation can make one better off without harming the other. However, it is not envy-free, since A envies B's comfortable shoes. Now, suppose they exchange only their left shoes; neither gains by the swap, but each now owns one comfortable and one uncomfortable shoe. The new allocation is envy-free but not Pareto optimal, since both would prefer a matching pair, even one of lower quality. This illustrates the tension between fairness (EF) and efficiency (PO): an allocation can be fair without being efficient, and vice versa.

2.3. Envy-freeness in practice: Experimental insights and applied contexts

One of the few experimental investigations into fair division is the study by Herreiner and Puppe (2009), which empirically tested the relevance of envy-freeness as a fairness criterion in the allocation of indivisible goods. Their laboratory experiments involved two- and three-person bargaining games, in which participants had to agree on a division of objects (and sometimes money) under time constraints. Crucially, participants were endowed with different preferences over the same objects, allowing the researchers to distinguish between intrapersonal fairness criteria (such as EF, based on one's own preferences) and interpersonal ones (such as inequality aversion or maximising the welfare of the worst-off).

The findings showed that while Pareto optimality and inequality aversion dominated participants' decisions, envy-freeness did exert an influence, particularly in cases where other criteria could not clearly determine a fair outcome. However, the role of EF appeared limited, possibly due to the experimental design: subjects did not receive the objects themselves but were instead paid in money according to the final allocation; what is more, the preferences were forced on players, which may have weakened the salience of envy between bundles.

Herreiner and Puppe's results draw an important distinction between intrapersonal and interpersonal fairness. The former refers to the absence of envy from the individual's own perspective ("I would not prefer your bundle over mine"), while the latter involves cross-personal comparisons of well-being ("You are better off than I am"). EF represents the intrapersonal notion, but it can indirectly contribute to interpersonal fairness when individuals interpret their allocations relative to their own expectations—a dynamic possibly linked to anchoring bias (Tversky & Kahneman, 1974). In that sense, allocations satisfying EF may also enhance perceived stability and acceptance over time, since agents who initially feel no envy are less likely to revise their fairness judgments later.

The limitations of incorporating subjective perspectives into fair division become evident in the study by Madevska Bogdanova and Simjanoska (2020), who applied envy-freeness to assess task allocation within complex collaborative projects. In their framework, agents' preferences could be objectively inferred from prior performance and task relevance; while this was successful, it also proved impossible to elicit subjective preferences that rendered the project feasible. Participants rejected assignments that they perceived as overly demanding or lacking prestige, regardless of their objective fit. Consequently, the authors concluded that purely preference-based formulations of EF may fail when participants' self-perceptions and social considerations strongly influence task acceptance; it is this problem that limits fairness criteria in general.

Another idea was devised for the school redistricting framework proposed by Procaccia et al. (2024) and offers another example of how EF interacts with other fairness notions in constrained environments. Here, the goal is to assign students to schools while respecting capacity and distance limits and maintaining demographic balance. Each group's utility depends on school quality—often represented by standardised test scores or ratings—and the algorithm seeks assignments that are envy-free across groups: no demographic group should prefer another group's school assignment. To achieve this, the authors' model combines graph-based optimisation (ensuring local balance and connectivity) with proportionality and Pareto constraints to avoid inefficiency. The inclusion of EF stabilises outcomes by reducing post-assignment discontent. Simulations show that while proportionality alone can equalise average utility, it does not prevent envy among groups assigned to lower-performing schools. Adding EF constraints, even approximately (via envy-minimising heuristics), yields allocations that are more politically and socially robust, as stakeholders perceive them as procedurally fair. In practice, this approach highlights EF's ability to legitimise complex institutional allocations where transparency and acceptance are as vital as efficiency.

One of the most compelling real-world implementations of EF principles can be found in the work of Budish and Kessler (2014), who tested the Approximate Competitive Equilibrium from Equal Incomes (CEEI) mechanism at the Wharton School of the University of Pennsylvania. The mechanism was introduced to replace a long-standing fake-money auction used to assign MBA students to courses. The prior system relied on students bidding with artificial points for limited course seats—a process that encouraged strategic manipulation and produced outcomes perceived as unfair and inefficient. By contrast, the CEEI algorithm simulates a competitive market equilibrium in which each student receives an equal (though slightly perturbed) budget and is allocated their most-preferred affordable bundle of courses given equilibrium prices. The experiment revealed that CEEI significantly reduced envy: only 31% of students displayed any envy under CEEI compared to 42% under the auction, and when assuming perfect preference reporting, the envy level dropped to just 4%. Moreover, the CEEI allocations were approximately Pareto efficient and incentive-compatible, encouraging truthful preference reporting. These findings confirmed theoretical predictions that EF-like outcomes can emerge in realistic, multi-object environments when combined with competitive equilibrium logic. The Wharton study also emphasised the importance of perceived fairness and stability in adoption. Students rated the new mechanism as fairer and simpler to use, and the administration ultimately replaced the old auction system with the CEEI-based software, branded Course Match. This case illustrates that EF—when integrated with equilibrium pricing and equal-income constraints—can move from abstract theory to practical institutional design, balancing normative fairness with computational feasibility and real-world acceptance.

2.4. Welfare and algorithms

Classical welfare economics traditionally evaluates outcomes through utilitarian or Pareto-based criteria, focusing on aggregate efficiency. The perspective adopted here follows a Rawlsian and egalitarian view (Rawls, 1971), where social welfare depends on how well off the least advantaged individuals are and on whether agents feel equally entitled.

Envy-freeness and its relaxations address welfare not through total utility, but through perceived fairness—a form of psychological welfare. In behavioural and experimental literature (e.g., Fehr & Schmidt, 1999; Herreiner & Puppe, 2009), the absence of envy is frequently interpreted as an operational indicator of fairness perception, since envy is one of the most direct behavioural manifestations of perceived injustice. This perception of fairness enhances:

- social stability—fewer grievances and higher acceptance of outcomes;
- procedural legitimacy—greater trust in the allocation mechanism;
- cooperation—improved compliance and participation in repeated or community-based settings.

Behavioural research (Fehr & Schmidt, 1999; Kahneman et al., 1986) confirms that individuals derive substantial welfare from fairness itself: they often prefer equitable processes to maximised payoffs. Consequently, transparent algorithms that are easy to understand—rather than merely optimal—can improve both perceived and experienced welfare.

Two algorithmic paradigms exemplify this balance between fairness and comprehensibility:

1. **Round-Robin Algorithm**
   In the round-robin procedure, agents take turns selecting items from the remaining pool according to a fixed or randomised order. Formally, for agents $N = \{1, \dots, n\}$ and goods M, each agent i selects their most valued remaining item in turn until all goods are allocated. This simple, strategy-proof procedure ensures (Budish, 2011; Plaut & Roughgarden, 2020):

   - EF1 under additive preferences—envy can be removed by removing one item from an envied bundle;
   - EFX when preferences are identical across agents. Its sequential nature and intuitive logic make it transparent and acceptable, even in non-cooperative settings, where comprehension and procedural trust are crucial.

2. **Envy-Cycle Elimination Algorithm (Lipton et al., 2004)**
   This algorithm constructs a directed envy graph where an edge $i \to j$ indicates that agent *i* envies agent *j*'s bundle ($v_i(A_i) < v_i(A_j)$). Whenever the graph contains a cycle, the algorithm reallocates bundles along that cycle to remove all involved envies simultaneously.

   The process repeats until no cycles remain. The final allocation is guaranteed to be EFX when agents valuations are additive and have the same order and EF1 and Pareto efficient under any additive valuations. Despite its algorithmic sophistication, its logic—eliminating "loops of unfairness"—is conceptually simple and aligns with human intuitions about balancing envy over time.

Transparency and accessibility is finely presented on spliddit.org, a public fair division platform developed by Goldman and Procaccia (2014). Spliddit offers users fair solutions to problems like rent division, inheritance, and credit sharing, using among others, the above proven algorithms.

For example:

- Its rent division module implements the market-based algorithm by Abdulkadiroğlu et al. (2004), which adjusts room prices until an EF and PO assignment is reached.
- The goods division module computes allocations that maximise each participant's guaranteed fairness level (EF, proportionality, or the Maximin Share).
- For credit division, Spliddit applies impartial rules that ensure each participant's share is determined solely by others' reports, ensuring procedural fairness.

Spliddit not only executes fair algorithms, it also explains them. Each application provides an accessible visualisation of how fairness guarantees are met, fulfilling its mission "to make the world a bit fairer" through education and transparency. As a large-scale, real-world implementation, Spliddit demonstrates how computational fairness mechanisms can enhance both perceived welfare and public trust—a combination rarely achieved by traditional welfare optimisation.

Conclusions

Envy-freeness remains one of the most compelling and conceptually elegant frameworks for fair allocation. While achieving exact EF is often infeasible in real-world settings, such as public housing, school assignments, or medical scheduling, it continues to provide a normative benchmark for fairness. Practical constraints, from budgetary or political limitations to behavioural factors such as loss aversion and reference dependence (Kahneman & Tversky, 1979), complicate implementation and acceptance. Yet EF and its relaxations offer a unique bridge between formal fairness and perceived justice.

EF ensures that no agent envies another's share, thereby establishing an objective and psychologically credible standard of fairness. Its strength lies not only in the allocation outcome but also in its procedural legitimacy—agents who perceive a process as fair are more likely to accept its results. However, EF frequently competes with efficiency: procedures that guarantee EF or near-EF outcomes are not always PO (Brams et al., 2013). For situation with indivisible goods, proposed relaxations, such as envy-freeness up to one good (EF1) and envy-freeness up to any good (EFX) have emerged as practical compromises. These refinements preserve the intuition of fairness while being computationally attainable through transparent methods like round-robin or envy-cycle elimination.

Empirical and algorithmic studies such as Spliddit.org, Wharton's Course Match, and school redistricting algorithms—demonstrate that adding EF solutions can work effectively when paired with clarity, transparency, and social context. Mechanisms grounded in EF are not only mathematically defensible but also socially sustainable, since they promote trust, compliance, and perceived legitimacy. This dual nature—rigorous yet intuitive—explains EF's enduring appeal across economics, computer science, and public policy.

Nevertheless, important challenges remain:

1. Fairness vs. efficiency: EF and Pareto-optimality often conflict; understanding when and how to prioritise one remains a central design question.
2. Preference elicitation: Real-world mechanisms must account for incomplete, noisy, or strategic preference reports without undermining fairness.
3. Dynamic fairness: Many allocation settings evolve over time; maintaining fairness across temporal changes requires new theoretical and algorithmic tools.
4. Strategic behaviour: EF assumes truthful reporting, yet agents may manipulate outcomes. Designing strategy-proof and envy-free mechanisms is still an open frontier.

In conclusion, envy-freeness and its variants offer a robust, theoretically grounded, and operationally feasible framework for equitable allocation. While they do not resolve every tension between fairness, efficiency, and strategic behaviour, they do provide the clear path toward stable, transparent, and trusted systems of distribution. Continued dialogue between theory, algorithms, and behavioural insight will be essential to adapt EF-based fairness to increasingly complex social and economic environments.

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