Damian Sołtysiak

The stability of the Keynes–Metzler–Goodwin monetary growth model

(Summary)

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The stability of the Keynes–Metzler–Goodwin monetary growth model The neoclassical growth theory based on the paradigm of dynamic general equilibrium and intertemporal utility maximization has been increasingly questioned since the global financial crisis of 2008. This prompts interest in other monetary models of economic growth which are more closely related to Keynesian economics. Of particular importance in this regard are the works of three Keynesian economists, Carl Chiarella, Peter Flaschel and Willi Semmler, who have been developing Keynesian economics over the last two decades, resulting in the joint publication (sometimes with other co-authors) of a series of books on Keynesian economics with an emphasis on interrelations between real and financial spheres of the economy. Notable works include the comprehensive monograph *The dynamics of Keynesian monetary growth* (Chiarella & Flaschel, 2000) and the fundamental trilogy *Reconstructing Keynesian* m*acroeconomics*, vol. 1-3 (Chiarella et al., 2012, 2013, 2014), in which these economist attempt to completely reinterpret and reconstruct Keynesian macroeconomics in its entirety[1]. Other important works by these authors include Chiarella et al. (2000); Asada et al. (2003); Chiarella et al. (2005), and Charpe et al. (2011).

Chiarella, Flaschel and Semmler's most important monetary growth model, which they analyzed in various forms, is "the Keynes-Metzler–Goodwin model" (abbreviated to the KMG model), named to emphasize its relationship to the concepts developed earlier by these three economists. The equations of the KMG model expressed ideas taken from Keynes's *General theory* (1936) and Goodwin's (1967) work on the interaction of growth and income distribution; these are the K and G components of the model. The KMG models take into account goods mar-

[1] Another alternative and important trend in economics relating to Keynes is post-Keynesian economics (Lavoie, 2014).

ket disequilibrium, the implication of changes in inventories and the gradual adjustment of inventories to its desired level. The dynamics of inventories are related also to the concept of expected sales, which is formulated according to Metzler's (1941) lines and, as such, constitutes therefore the M-component of the KMG approach. To the same extent, the KMG model also relates to the Keynes-Wicksell-type models presented in Stein (1966), Rose (1966) and Fischer (1972), among other authors. In addition, its similarity to the Keynesian model presented in Sargent (1987, Ch. 5) should be noted.

All versions of the KMG model describe dynamics in the economy through a complex system of non-linear differential equations that reflect adaptive decision-making processes. The KMG model describes an economy consisting of the household sector, the government sector and firms. The level of consumption depends on the dynamics of employment and the dynamics of prices and wages described by two separate Phillips curves (for prices and wages). In turn, investments depend on factors such as the variation over time of the difference between the return on real capital and the real interest rate, as well as the degree of capacity utilization. Actual and expected inventories play a key role in this model. Companies adjust production levels based on their expected sales and changes in inventories, which in turn affects production capacity utilization rates and employment levels. The economy's production capacity depends both on the dynamics of physical capital and labor productivity growth.

The KMG model continues to be extended and modified; one example is Ogawa (2019a, 2019b, 2020), who extended the KMG model to a two-sector model; another is Flaschel (2020), in which an extensive KMG model is used to analyze how taxes, transfer payments and government spending improve the social protection of the employee household sector. Chiarella et al.'s (2021) latest research (authored by Peter Flaschel, Reiner Franke and the late Carl Chiarella) marks the culmination of the development of the Bielefeld school of macroeconomic thought.

Asada et al. (2018, 2019) recently presented an interesting variant of the KMG model that features private corporate debt to model the effects of fiscal and monetary policies. These authors also deal with the existence of limit cycles in the KMG model. Keynesian monetary growth models similar to the KMG model, in which limit cycles or periodic orbits may exist, also appear in recent words by Murakami (2016, 2018, 2020) and Araujo et al. (2020).

When analyzing KMG models, as in any economic growth theory, two research problems come to the fore. The first concerns whether the economy described by a given model can evolve sustainably along a balanced growth path. Since balanced growth refers to the growth in which proportions between variables (a structure of the economy) remain constant, whether such a balanced growth path exists comes down to the existence of such proportions between variables that facilitate such growth. In the terminology of economic growth theory, this concerns the existence of a steady state. In monetary growth models, such as KMG models, an interesting issue is the steady state's dependence on or a lack of independence of monetary policy, in particular, concerning money supply growth (the problem of money neutrality). The second key research problem concerns whether an economy in its initial state and not on a balanced growth path (and thus not in a steady state) converges to this path over time, reaching proportions close to those characterizing a steady state. If such a convergence occurs, we speak of an asymptotic stability of the steady state. Stability is a desirable feature in mathematical models of economic growth because it indicates that in the course of time the growth of a modeled economy becomes increasingly harmonious. A lack of stability is a serious shortcoming of an economy described by such a model, as it implies the existence of a mechanism that deepens imbalances, which can lead to an economic crisis.

It should be emphasized that stability is not an unconditional feature of a model. Mathematical theorems concerning the stability of various growth models always rely on a number of assumptions essential to proving stability. In the case of KMG models, the assumptions that guarantee the stability of an economy, those concerning money supply growth or the rules of interest rate formation are of particular interest. Such assumptions make it possible to formulate conclusions as to whether and how monetary policy can affect the steady state and the stability or instability of an economy.

This dissertation constitutes an original contribution to the research on KMG models. It two proposes and analyzes new versions of the KMG model. While having many features in common, these models differ fundamentally in the way they describe the money market. The first model (referred to in the monograph as the KMG model) assumes exponential money supply growth. At the same time, the demand for money is an increasing function of the expected final demand in the product market and a decreasing function of the interest rate. Consequently, assuming money market equilibrium, the interest rate must adjust to changes in expected demand in such a way that the demand for money (equal to the money supply) also grows exponentially.

In the second model, a money demand function does not appear at all, and the dynamics of the interest rate are described by a Taylor equation (hence the second model is called the KMGT model). According to the Taylor equation, changes in the interest rate depend on three factors: a deviation in the current interest rate from its desired value, a deviation in the inflation rate from the desired inflation rate (inflation target), and a deviation in the capacity utilization ratio from its natural level. In the KMGT model, changes in the money supply are determined ex post that depends mainly on the demand of businesses for investment loans and the state's borrowing needs (bond issuance).

Both models are the result of several modifications the author introduced into standard KMG models. A point of reference for the first model presented in the monograph is the KMG model developed by Asada et al. (2003), and for the second of the models, the KMG model by Charpe et al. (2011)

Another important difference between the models in question is that the KMG model features only a behavioral equation describing factors affecting the level of net investment, with no explicitly indicated sources of financing for these investments. In addition to the aforementioned behavioral equation (in an expanded form), the KMGT model also features an equation showing directly that companies' investments are financed by their profits and investment credit.

In the monograph, four aims have been achieved. The first was to develop two new KMG-type models differing from the versions analyzed in the existing literature by virtue of several modifications. The second aim was to transform the formulated models into an intensive form and to determine and analyze their steady states, corresponding to balanced growth paths. The most important goal in the entire dissertation, and the most difficult to achieve, was to prove the stability of the two KMG models studied. The last of the aims was to conduct a series of computer simulations using data on the Polish economy. The simulations successfully provided answers to several questions that could not be obtained through qualitative model analysis.

The modifications introduced aim to improve the model by eliminating some crude and questionable elements from its equations. These elements seem to have been selected not for their economic relevance but primarily for their simplicity, which facilitates mathematical analysis of the model. The first two modifications concern equations describing determinants of the growth rate of fixed capital $\dot{K}/K$ and the growth rate of expected demand $\dot{Y}^e/Y^e$. In Chiarella et al.'s KMG models presented above, the equations contain parameter n, added simply to other components. Depending on the particular version of the model, n expresses either the constant growth rate of labor supply or the growth rate of labor productivity, or the sum of these two growth rates. Adding n to these equations is extremely convenient since, in the steady state, the remaining components of the equations become zero, which implies immediately that in this steady state both fixed capital and expected demand grow at rate n. In the present monograph, the dynamics of fixed capital and expected demand are much more thoroughly elaborated. In the equation describing $\dot{K}/K$, the constant parameter n has been replaced by a variable expressing the growth rate of expected demand $\dot{Y}^e/Y^e$. At the same time, parameter n has also been removed from the equation for $\dot{Y}^e/Y^e$ and substituted with the growth rate of the real wages as one of the two factors influencing $\dot{Y}^e/Y^e$, which is economically much more justifiable. The growth rate of real wages is an endogenous variable that depends on other model variables – principally, on the growth rate of the nominal wage and the inflation rate. The third modification is similar in character and concerns the equation showing three factors determining output level Y. In this case, one doubtful factor (component) nN^d (N^d – desired level of inventories) has been replaced with $\dot{Y}^e$, i.e. the change in expected demand.

Further important modifications concern the assumptions regarding taxes and real interest on bonds. In all versions of the KMG model presented by Chiarella et al., derivation of the intensive form of the model is facilitated by assuming that real taxes net of interest are assumed to be constant per unit of capital. As a consequence, the dependence of taxes on tax rates imposed on labor or capital incomes is not taken into account. Such dependence is considered only in the present model, which allows for a more comprehensive analysis of fiscal policy regarding taxes and the government budget deficit. In particular, in contrast to previous models, the ratio of government budget deficit to fixed capital ceases to be constant in time, despite the assumption of a constant ratio of government spending to capital.

The above modifications apply to both models presented in the monograph. Another modification applies only to the KMG model from the first chapter and concerns introducing a non-linear money demand function into this model. The authors of earlier versions of KMG models found in the literature consider a linear money demand function, whose values depend on expected demand and on the current interest rate deviating from a steady state interest rate, which is known in advance.[2] As a consequence, money demand in the steady state is determined exclusively by expected demand, independent of the value of the exogenously determined steady-state interest rate. This also raises doubts. Therefore, in the model proposed and presented in the monograph, the demand for money is described by a nonlinear function that depends on the expected demand and the actual, endogenously determined interest rate. As a result, the interest rate in the steady state becomes an endogenous value that depends on all parameters in the model.

The aforementioned modifications result in the appearance of new feedback and increased complexity in the two models analyzed. Nevertheless, it transpires that the models presented in the monograph can be transformed into an intensive form and the steady state can be uniquely determined, which is the second goal of the monograph. This opens the way for the monograph's third aim, namely, demonstrating the local asymptotic stability of the models in question.

Determining the steady state enables us to analyze the asymptotic stability of the KMG model. This consists in proving the convergence of variables of the intensive form model toward their steady-state values when time tends to infinity. Such steady-state stability is equivalent to the ability of the economy to converge toward a balanced growth path. Investigating such a convergence is the focus of interest of any mathematical theory of economic growth, be it Keynesian or neoclassical.

The intensive form model is a system of nonlinear differential equations, thus, to prove its local asymptotic stability, it must be shown that all of the eigenvalues of the Jacobian matrix of this system are either negative numbers or complex numbers with

[2] Chiarella et al. (2000, p. 279) admit that the above assumption was introduced in order to ease an analysis of a model.

negative real parts. Investigating these eigenvalues is a standard method for proving stability, which has been applied hundreds of times to different dynamic models. Since eigenvalues are also roots of the corresponding polynomial, their analyzing them is relatively simple when dealing with a system of two differential equations; however, it poses a significantly more difficult and sophisticated challenge with high-dimensional systems such as the KMG model. In the latter case, it is likely that the only way to show that the eigenvalues of a given Jacobian matrix guarantee stability is by subsequent zeroing of appropriate matrix parameters, which enables a multiple Laplace expansion of the determinant of the Jacobian matrix in order to obtain a sequence of matrices of a progressively descending order, whose eigenvalues can be more easily analyzed. First, the lowest-order matrix is shown to have appropriate eigenvalues. Subsequently, through a subsequent restoration of small positive values of previously zeroed parameters, it is demonstrated that the original Jacobian matrix also has eigenvalues which are either negative numbers or complex numbers with negative real parts.

The first rigorous proof of stability of the 7-dimensional KMG model, based on the idea outlined above, was presented 20 years ago in Chiarella et al. (2002). Other versions of the proof of stability (concerning modified KMG models) can be found in Asada et al. (2003) and Chiarella et al. (2006). In the latter publication, the authors use the term *cascade of stable matrices approach* to express the general idea underlying this proof.

While the same idea is also used for proof of stability of the KMG model presented in this monograph, this does not imply that it is merely a re-hash of earlier proofs. The modifications introduced into the model and the resultant new loops render its mathematical structure more complicated. This is particularly visible in the intensive form of the analyzed model, for which the Jacobian matrix is determined, and whose eigenvalues are examined. This is the first reason for which the proof presented differs from the earlier versions. The second reason is that the proof is obtained under different assumptions about model parameters, including tax rates, which are not present in the models analyzed by Chiarella et al. The third reason concerns certain differences in reasoning and denotation.

To prove stability, two fundamental difficulties had to be overcome. The first concerned the values of partial derivatives, evaluated at the steady state, of all functions defining the dynamics of the intensive form model; these derivatives constitute elements of the Jacobian matrix. The second difficulty concerned which parameters, and in which sequence, should be zeroed to show conclusively that all eigenvalues of the Jacobian matrix are either negative or complex numbers with negative real parts. However, there was no ready answer to or even a suggestion about how to resolve this crucial question. Hence, a laborious heuristic process was necessary, involving a series of attempts that ultimately proved successful.

Proving the existence of a steady state and demonstrating that under certain assumptions this state is locally asymptotically stable does not give an answer to many economically important questions about the development of an economy described by the KMG and KMGT models discussed above. One such question concerns how long it takes for the economy, starting from an initial state, to assume a path of balanced growth. Another equally important question is how far away from a steady state the economy's initial state can be for it to converge. These and many other questions can only be answered by means of computer simulations. Conducting such simulations was the fourth and final aim of the monograph.

Simulations were conducted for data on the Polish economy using both models analyzed in the dissertation. Before starting the simulations, the models were modified to maximize the information about the Polish economy that could be included, while preserving the mathematical structure. Then, based on the same data, the models' parameters were calibrated, and steady states of the models were determined.

The results of the numerous computer simulations confirmed the validity of the theorems proved in the monograph regarding the local asymptotic stability of the steady state of the KMG and KMGT models. The simulation results were shown to be similar to those obtained by other authors and confirmed that the economy's arrival at the steady state is slow, oscillatory, and requires a very long time horizon. In the simulation results, the steady state appears as a one that, although unattainable in a realistic time horizon, acts as a "compass" for the economy in determining the direction of structural changes caused by market processes. The practical importance of the stability of KMG models is thus demonstrated.

In addition to empirical stability analysis using computer simulations, the short-term impact of fiscal and monetary policies on the economy was also analyzed. The simulation results from the KMG model and the KMGT model were compared. In addition, simulations were run on the efficacy of monetary policy shielding measures against economic disruptions related to the SARS-CoV-2 virus pandemic. The analysis focused on how an expansionary monetary policy can mitigate the effects of these perturbations and evaluated the long-term effects of this.

The monograph is divided into four chapters. In the first chapter, the author's version of the KMG model with exponential money supply growth is presented. This model is then transformed into its intensive form, the steady state is determined, and the sensitivity of the steady state to changes in model parameters are examined.

Chapter Two introduces another version of the KMG model, incorporating the Taylor rule and investment credit, after which this model is compared with the model presented in Chapter One.

In the third chapter, mathematical proof of the stability of the two models described in the previous chapters is presented. This proof constitutes the main theoretical results demonstrated in the author's dissertation. The proven stability theorems provide sufficient conditions for the convergence of the modeled economies to the balanced growth path.

The fourth chapter is devoted to computer simulations based on data from the Polish economy and using the models studied. The results of these simulations confirm the validity of the stability theorems proven in Chapter Three.

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References

Araujo, R., Flaschel, P., & Moreira, H. (2020). Limit cycles in a model of supply-side liquidity/profit-rate in the presence of a Phillips curve, *Economia, 21*(2), 145-159. https://doi.org/10.1016/j.econ.2019.09.003

Asada, T., Chiarella, C., Flaschel, P., & Franke, F. (2003). *Open economy macrodynamics*. Springer.

Asada, T., Demetrian, M., & Zimka, R. (2018). On dynamics in a Keynesian model of monetary and fiscal policy with debt effect. *Communications in Nonlinear Science and Numerical Simulations, 58*, 131-146. https://doi.org/10.1016/j.cnsns.2017.06.013

Asada, T., Demetrian, M., & Zimka, R. (2019). On dynamics in a Keynesian model of monetary and fiscal stabilization policy mix with twin debt accumulation. *Metroeconomica*, *70*(3), 365-383. https://doi.org/10.1111/meca.12220

Charpe, M., Chiarella, C., Flaschel, P., & Semmler, W. (2011). *Financial assets, debt and liquidity crises: A Keynesian approach*. Cambridge University Press.

Chiarella, C., & Flaschel, P. (2000). *The dynamics of Keynesian monetary growth*. Cambridge University Press. https://doi.org/10.1017/CBO9780511492396

Chiarella, C., Flaschel, P., & Semmler, W. (2012). *Reconstructing Keynesian macroeconomics*, vol. 1: *Partial perspectives*. Routledge.

Chiarella, C., Flaschel, P., & Semmler, W. (2013). *Reconstructing Keynesian macroeconomics*, vol. 2: *Integrated Approaches*. Routledge.

Chiarella, C., Flaschel, P., & Semmler, W. (2014). *Reconstructing Keynesian macroeconomics*, vol. 3: *Macroeconomic activity, banking and financial markets*. Routledge.

Chiarella, C., Flaschel, P., Groh, G., & Semmler, W. (2000). *Disequilibrium, growth and labor market dynamics*. Springer.

Chiarella, C., Flaschel, P., Franke, R., & Semmler, W. (2002). *Stability analysis of a high--dimensional macrodynamic model of real-financial interaction: A cascade of matrices approach*. Working Paper Series 123. Finance Discipline Group, UTS Business School, University of Technology, Sydney.

Chiarella, C., Flaschel, P., & Franke, R. (2005). *Foundations for a disequilibrium theory of the business cycle. Qualitative analysis and quantitative assessment*. Cambridge University Press. https://doi.org/10.1017/CBO9780511492402

Chiarella, C., Flaschel, P., Franke, R., & Semmler, W. (2006). A high-dimensional model of real-financial market interaction: The cascade of stable matrices approach. In C. Chiarella, R. Franke, P. Flaschel, & W. Semmler (Eds.), *Quantitative and empirical analysis* *of nonlinear dynamic macromodels, contributions to economic analysis* (pp. 359-384), Elsevier. https://doi.org/10.1016/S0573-8555(05)77011-5

Chiarella, C., Flaschel, P., Franke, R., Araujo, R., Charpe, M., Proaño, Ch., & Szczutkowski, A. (2021). *Unabalanced growth from a balanced perspective*, Edward Elgar.

Fischer, S. (1972). Keynes–Wicksell and neoclassical models of money and growth. *American Economic Review*, *62*, 880-890.

Flaschel, P. (2020). A baseline model of 'Social Protection' in open economies of the KMG variety. Bielefeld. https://doi.org/10.4119/unibi/2942156

Goodwin, R. M. (1967). A growth cycle. In C. H. Feinstein (Ed.), *Socialism, capitalism and economic growth* (pp. 54-58). Cambridge University Press.

Keynes, J. M. (1936). *The general theory of employment, interest and money*. Macmillan.

Lavoie, M. (2014). *Post-Keynesian economics: New foundations*, Edward Elgar.

Metzler, L. A. (1941). The nature and stability of inventory cycles. *Review of Economic Statistics*, *23*(3), 113-129. https://doi.org/10.2307/1927555

Murakami, H. (2016). Alternative monetary policies and economic stability in a medium-term Keynesian model. *Evolutionary and Institutional Economics Review*, *13*(2), 323-362. https://doi.org/10.1007/s40844-016-0045-2

Murakami, H. (2018). Existence and uniqueness of growth cycles in post Keynesian systems. *Economic Modelling*, *75*, 293-304. https://doi.org/10.1016/j.econmod.2018.07.001

Murakami, H. (2020). Monetary policy in the unique growth cycle of post Keynesian systems. *Structural Change and Economic Dynamics, 52*(C), 39-49. https://doi.org/10.1016/j.strueco.2019.10.002

Ogawa, S. (2019a). Dynamic analysis of a disequilibrium macroeconomic model with dual labor markets. *Metroeconomica, 70*(3), 525-550. https://doi.org/10.1111/meca.12255

Ogawa, S. (2019b). *Effective demand and quantity constrained growth: A simple two sector disequilibrium approach*. MPRA Paper, 93336. https://mpra.ub.uni-muenchen.de/93336/

Ogawa, S. (2020). *Monetary growth with disequilibrium: A non-Walrasian baseline model.* MPRA Paper, 101236. University Library of Munich, Germany.

Rose, H. (1966). Unemployment in a theory of growth. *International Economic Review*, *7*(3), 260-282. https://doi.org/10.2307/2525525

Sargent, T. (1987). *Macroeconomic theory* (2nd ed.). Academic Press.

Stein, J. (1966). Money and capacity growth. *Journal of Political Economy*, *74*(5), 451-465. https://doi.org/10.1086/259199

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