AUTOMATIC IDENTIFICATION OF TURNING POINTS WITH HMM-BASED INDICATOR

https://doi.org/10.18559/978-83-8211-129-3/3

Michał Bernardelli

Collegium of Economic Analysis, SGH Warsaw School of Economics

Abstract

In this paper, the issues of constructing early warning indicators, as well as the business cycle turning point detection, are raised. Following the original procedure presented by Bernardelli (2020), modification and simplification of one of the procedure stages were proposed. The stage with turning point identification was replaced by a combination of well-known and recognised asymmetric Christiano-Fitzgerald filter and the Bry-Boschan routine of cyclical turning point selection. Presenting improvement in the procedure was the key objective of this article. The basic idea of the remaining part of the HMM Signature Indicator procedure lies in the definition of the signature, which captures the pattern in the considered time series, just before the peak or trough. In the presented approach, the following concepts were exploited: the hidden Markov model (HMM), Viterbi path (VP) and Monte Carlo simulations (MC).

However, the value-added characteristic of this article is not only the proposition of a new, highly parametrisable method for quantitative representation of the business cycle. An integral part of the research was the validation of the presented approach. The data from the business tendency survey in the Polish manufacturing industry conducted by RIED SGH was used to design 2 different HMM Signature Indicators. Each of the them was constructed on the basis of different input time series. Despite this, both demonstrated leading properties, and therefore, effectiveness of the proposed solution was proved. In the majority of cases, the turning points were properly caught in reasonable advance. This means that the presented method can be successfully used for the construction of early warning indicators.

Keywords: indicator, hidden Markov model, Viterbi path, turning point, business tendency survey, Christiano-Fitzgerald filter, Bry-Boschan algorithm.

JEL codes: C63, C88, E37.

Introduction

Assessment of the past and current economic situation is not straightforward and could vary depending on the approach used for the identification of business cycle phases. Within such a context, predicting the future state of the economy is even

Suggested citation:
Bernardelli, M. (2022). Automatic identification of turning points with HMM-based indicator. In S. Białowąs (Ed.), *Economic tendency surveys and economic policy—measuring output gaps and growth potentials* (pp. 39–55). Poznań: Poznań University of Economics and Business Press. https://doi.org/10.18559/978-83-8211-129-3/3

more difficult. Over the years, many methods of analysis have been developed to identify a business cycle change at the earliest possible moment (Boldin, 1994). Therefore, all the methods have one thing in common: the goal to monitor the economic situation and provide some warning signals in advance. Having information about the impending recession allows to take reduction measures with regard to damage caused by the crisis or even avoid a shift towards a phase of recession.

A considerable percentage of monitoring methods are based on the construction of the indicator. The proper indicator should have the feature of signalling changes in the economy early enough for policymakers to take the necessary action. The process of constructing a new indicator or adaptation of existing ones requires a great deal of knowledge and effort. There are many aspects to consider, including selection and transformation of input data, choice of modelling method, along with parameter calibration and extensive testing based on historical data. The universal approach presented by Bernardelli (2020) allows to address almost the complete indicator construction process, apart from the input data selection. This so-called HMM Signature Indicator procedure explores the advantages of the hidden Markov models (HMM), Viterbi paths (VP), and the idea of the signature, discussed in more detail later in this article. A few drawbacks have been identified concerning this approach, particularly, at the last stage of the algorithm, where the turning points are established on the values of the indicator. While the proposed procedure is methodologically correct and seems to work in practice, it is also quite complex and difficult to execute. Therefore, a modification of the procedure was proposed, in which the final stage was simplified by the use of well-known and recognised methods for time series decomposition and the Bry-Boschan routine of cyclical turning points selection (Bry & Boschan, 1971). Presenting improvement in the procedure was the key objective of this article. A detailed description was given along with a broad discussion on the limitations and possibilities of generalisation regarding this approach.

One of the more known indicators is the OECD composite leading indicator (CLI; OECD, 2022), designed to capture large fluctuation of economic activity in advance. There are, however, many country-specific indicators, which even better reflect the economic specifics of a particular country. Some of those indicators are based on macro-economic data, while some allow to explore the benefits of business tendency surveys. In Poland, a monthly business tendency survey is conducted, for example, by the widely-recognised Research Institute for Economic Development (RIED) at SGH Warsaw School of Economics. Based on the opinions of the respondents from the manufacturing industry (see Adamowicz, Dudek, Kluza, Ratuszny, & Walczyk, 2019), the industrial confidence index (RIED ICI) is calculated, which is proved to be a leading indicator in relation to the economic situation of the whole Polish industry. A secondary research goal was empirical verification regarding the effectiveness of this new procedure, based on the data from the business tendency survey carried out in the Polish manufacturing industry. The obtained results were compared with the reference time series for which the RIED ICI was chosen. The selection was made because this indicator, and the related turning points, are determined on the basis of the same input data, i.e. a business tendency survey.

This paper consists of 5 sections. After the Introduction, in section 1, basic facts about HMMs and VPs are presented. Both are the main concepts used in the HMM Signature Indicator procedure. These concepts are introduced not in the strict mathematical notation, but rather from an intuitive point of view in a degree allowing to understand the HMM Signature Indicator procedure, which is described in section 2. The differences between the original (Bernardelli, 2020) and the proposed procedure are emphasized. The empirical analysis, which is an implementation of the indicator construction process, is demonstrated in section 3. In this part, the characteristics of selected questions from the business tendency survey are presented. Also, a tabular summary and visualisation of the results are given. The article ends with concluding remarks and ideas for further research in this area.

1. HMMs and VPs concepts

In this section, the hidden Markov model concept is introduced. However, focus is shifted towards the idea behind the theory, instead of presenting mathematical details and formal definitions. The description has been enriched with examples and figures. The description will begin with a historical outline and an overview of the known applications.

HMMs are related to the name Hamilton (1994), and are a generalisation of the Markov models. In 1970, the algorithm of HMM parameters estimation was proposed (Baum, Petrie, Soules, & Weiss, 1970). This algorithm is known by the name of Baum-Welch. In the HMM, compared to the classical models, the—included in the name—hidden layer is added (Cappé, Moulines, & Rydén, 2005). Therefore, the model consists of two layers: one with observable observations, based on which the path of states from the second, unobservable layer is discovered. Discovering comes down to identifying the pattern to which the input data is best suited (of course, if such a pattern exists). The idea behind the HMM is in line with pattern recognition and, therefore, the application occurs in all areas where these kinds of processes are explored, such as speech, handwriting, gesture or voice recognition. Recent years have seen an increase in the application of macro-economics. HMM has proved to be useful in, e.g. business cycles synchronisation analysis (Smith & Summers, 2005; Dufrénot & Keddad, 2014), turning point identification (Chauvet & Hamilton, 2005; Bernardelli, 2015) or convergence analysis (Bernardelli, Próchniak, & Witkowski, 2021). This method was also the basis for constructing early warning indicators (Abberger & Nierhaus, 2010). In all of those fields and applications, HMMs proved to be an excellent alternative to the classical methods.

The key advantage of this approach is the nearly total lack of assumptions to be fulfilled, compared to econometric methods. However, there are 2 major complications related to the HMM. The first one is the potential unoptimality of the result. The Baum-Welch algorithm is a deterministic method, but the resulting set of model parameters strongly depends on the initial values. Therefore, beginning with arbitrarily chosen initial values, we can obtain a solution that will not be optimal. The easiest way to increase the chance of finding an optimal solution is to repeat the computations for different initial points. This kind of repetition for random initial values is referred to as Monte Carlo simulation (Cappé et al., 2005). The number of simulations depends on the stability of the results. It needs to be emphasized that this non-deterministic nature regarding the combination of the Baum-Welch algorithm and Monte Carlo simulations does not make the results probabilistic because the simulation does not concern the input data but the initial points. Hence, non-determinism means suboptimalism of the result.

The second issue concerns the output format of the Baum-Welch algorithm, which is just a set of probabilities of being in a particular state at each time point. Thus, the output set of probabilities needs to be assigned to the correct order of states. Several possible solutions to this problem exist, but in practice, smoothed or filtered probabilities were the most commonly used in the early years (Milas, Rothman, Dijk, & Wildasin, 2006). However, a superior approach exists, based on the completely deterministic Viterbi algorithm (Viterbi, 1967). In this approach, the whole period covered by the analysis is taken into consideration at once, instead of "step by step" decoding. It is much more computationally complex, but results in the most likely sequence of hidden states called the Viterbi path.

In the HMM Signature Indicator procedure, described in the next section, any hidden Markov model may be used, regardless of the number of states or the assumed form of the observable layer probability distribution. With regard to the empirical analysis presented in this article, Gaussian distribution was assumed, and the 3-element state-space $S = \{0, 1/2, 1\}$. Interpretation of states depends on context, but in traditional notation, state 1 is always associated with periods of relatively good conditions, while state 0 is associated with a worse situation. State 1/2 corresponds to an uncertain, transient period. The ordering of states is forced by assuming that state 1 is associated with the greatest mean value (within the meaning of the parameter with normal distribution), and state 0 with the smallest mean value.

To visualise the effectiveness of pattern recognition with the use of HMM, in Figure 1, raw data from the RIED business tendency survey for the question

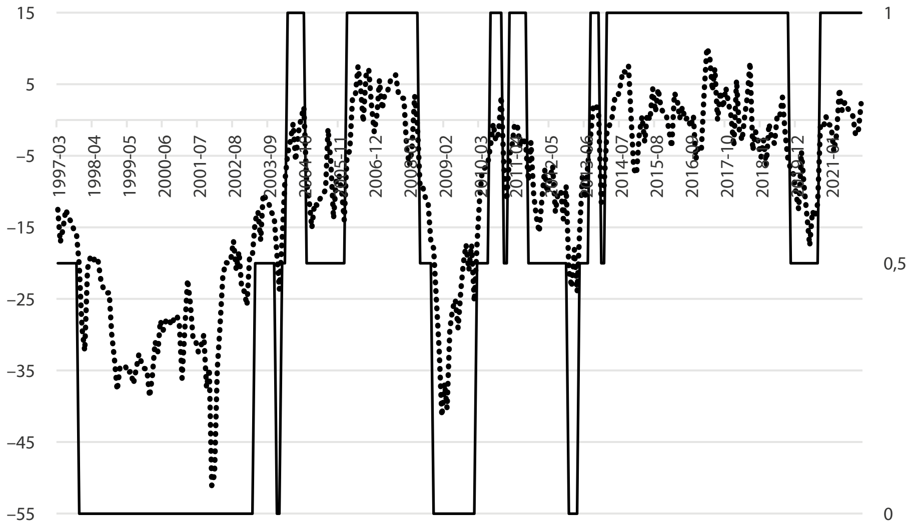

Figure 1. The difference between percentages of positive and negative answers to the question from the RIED business tendency survey about the level of employment in the Polish manufacturing industry (dotted line), along with the 3-state VP (continuous line) from March 1997 to December 2021
Source: Own calculation based on RIED data.

about the level of employment is given along with the 3-state VP. The details about the RIED surveys and RIED ICI are described in the next section. Here, only the visualisation of an exemplary HMM usage is presented as an argument for the suitability of this approach. All phases detected by the method based on the HMM and VP coincide with the values of the time series: for peaks states, 1 or 1/2 are assigned, and with a phase of lower level of employment states, 0 or 1/2 are associated. In general, 3-state HMMs give more accurate results compared to the 2-state HMMs. Adding more states is possible, but at the cost of greater computational complexity, lower stability of solutions and more difficult interpretation of the results. A maximum number of states is also strictly related to the length of the input time series.

The VPs for other, selected questions from the business tendency survey are presented in the Appendix. Selected interdependencies can be observed for answering some questions, but the VPs show that the questions carry significantly different information. An appropriate combination of them into an indicator is possible, as evidenced by the existence of the RIED ICI, which proves to be of leading nature. In the section with empirical analysis, alternative indicators are shown. They are based on the same input data, but a different construction methodology.

2. Construction of the HMM Signature Indicator

In this section, the procedure of the construction of the HMM Signature Indicator is given. Based on the description of the original procedure (Bernardelli, 2020), the stages of the algorithm will be presented, where the last stage is the added value of this article. This stage is changed compared to the original formulation. The whole procedure is split into 3 separate stages. The first one is concentrated on the construction of the reference signature, the second on the construction of the HMM Signature Indicator, and the third, on turning point identification.

At the beginning of the description regarding this procedure, the definition of the signature will be given. Let us assume that the VPs have been computed for the set of time series. Afterwards, the arithmetic means of states from VPs at each time point from the given set of points is calculated. The vector consisting of these arithmetic means is called a signature, while the length of the vector is called the size of the signature.

Stage 1 of the procedure concerns constructing the reference signature. The idea is to capture the pattern in the considered time series just before the peak or trough. For this to be achieved, some identification of historical turning points is needed. Based on those points, the steps of reference signature construction are as follows:

1. Select the set of time series. That set is an input, after which transformations become a component of the signature.
2. For each of the selected time series, perform the procedure described in (Bernardelli et al., 2021), using the HMM, VP, and Monte Carlo simulations to obtain the set of states. During this step, the number of states must be fixed. In the empirical analysis, for each of the input time series, 10,000 simulations are performed.
3. At each time point, calculate the arithmetic mean of states associated with this particular time point from every time series in the set.
4. Let n be the chosen size of the signature. Based on the reference set of turning points, calculate the reference signature that represents the image of the studied phenomenon (e.g. economic climate) observed at the turning point and n time points prior to this turning point. The output of this step is a vector with the length of $n + 1$, being a reflection of the situation directly before (n observations) and during the potential turning point. This vector is called a reference signature.

From the steps of the 1st stage of the procedure, it follows that to define a signature, it is necessary to set 3 parameters: the number of variables included in the signature, the size of the signature and of the HMM state space. To construct the reference signature, in addition, a set of reference turning points is needed.

At the end of the 1st stage, the reference signature is known, as well as the signatures at every time point, except for the first *n* observations.[1]

Stage 2 of the procedure leads to the construction of the HMM Signature Indicator. The values of this indicator are specified for each time point, apart from the first *n* points. At each time point, the signature is calculated and compared to the reference signature. The value of the indicator is only the value of the distance measure between these signatures. In particular, in the empirical analysis presented in this article, the root mean square error (RMSE) is used. The interpretation is straightforward, the RMSE measure indicates how similar the current situation (expressed in the form of a signature) is compared to the reference signature. The smaller the RMSE measure, the more similar the signatures, and the higher probability of being at a turning point.

Stage 3 of the procedure is the transformation of the calculated indicator into a set of turning points. This is the part of the procedure that differs from the original formulation, where the proposition of a new algorithm for turning point identification, based on the given time series, is given. This algorithm, however, turns out to be unnecessarily complex. Due to the fact, that this stage cannot be omitted, and the HMM Signature Indicator will probably not be a smooth function (or rather too volatile), existing algorithms cannot be applied directly or—to be more precise—the algorithms would identify too many turning points to be useful from a practical point of view. Therefore, a modification of this stage of the procedure was proposed. A detailed description can be summarised in the following steps:

1. Perform decomposition of the indicator into the cyclical and trend components. There are a few ways to achieve this goal, e.g. Seasonal and Trend decomposition using Loess (STL, see Cleveland, Cleveland, McRae, & Terpenning, 1990), Tramo-Seats decomposition (Dagum & Bianconcini, 2016), or X-13-ARIMA-SEATS decomposition (Sax & Eddelbuettel, 2018). In this procedure, the asymmetric Christiano-Fitzgerald filter (Christiano & Fitzgerald, 2003; Nilsson & Gyomai, 2011) of a time series was used. The following parameter values have been established:
   - 24 as a minimum period of oscillation for the desired component;
   - 144 as a maximum period of oscillation for the desired component;
   - drift and unit root in time series assumed.

   The cyclical component of the time series is passed on to the next step of this stage.
2. Use the Bry-Boschan routine (Bry & Boschan, 1971) to identify turning points in the cyclical component of the HMM Signature Indicator. This routine finds local extrema in the cycle series, ensuring alternating peaks and troughs, as well as phase and cycle length constraints. This method is

[1] For such observations, the signature can be calculated, but not for the whole range of times.

widely used, e.g. OECD applies a simplified version of the original Bry and Boschan routine, starting from December 2008 (see Federal Reserve Bank of St. Louis, n.d.).

A cyclical component of the exemplary HMM Signature Indicator (step 1) with turning points (step 2) is given in Figure 2. The performed computations resulted in the HMM Signature Indicator denoted by the HMM IND 1, which is described in the next section.

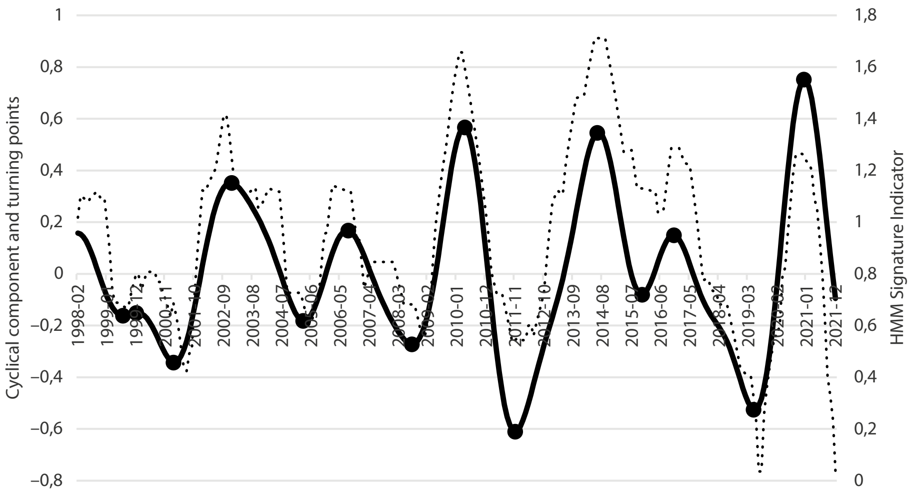

Figure 2. The HMM IND 1 (dotted line) along with the cyclical component (continuous line) and turning points (points) from March 1997 to December 2021
Source: Own calculation.

The proposed procedure of computing signatures, indicators and related turning points is a simplified and better methodologically justified variant of the original procedure. The modification involves the use of broadly-recognised decomposition methods and the Bry-Boschan algorithm. The procedure, as a whole, could be treated as a universal template to construct new indicators. The illustration of this procedure and the verification of leading properties are given in the subsequent section.

3. Empirical analysis

This section consists of 2 parts. In the first one, data characteristics are given, while in the second part, the construction of 2 exemplary HMM Signature Indicators is described. This section ends with a brief discussion of the results and a comparison with the reference RIED ICI time series.

Data used in the empirical analysis come from the business tendency survey in the Polish manufacturing industry, conducted monthly by the RIED. In this survey,[2] respondents evaluate changes in selected areas of economic activity, by answering 8 questions (in 8 versions: retrospective about the current situation, and perspective about the future):

- volume of production (prod);
- volume of total orders (order);
- volume of export orders (order);
- finished goods inventories (stock);
- selling prices of products (price);
- level of employment (employ);
- financial standing (fin);
- general economic situation in Poland (gen).

For every question, balances, as the difference between percentages of positive and negative answers, are calculated. An example of a time series comprising such balances is given in Figure 1 (for the question about the level of employment). Graphs of time series regarding stock, price, fin and gen, are shown in the figures included in the Appendix.

Since January 2013, the RIED ICI has been calculated as an arithmetic mean of 3 balances: production expectations, the current volume of total orders and finished goods inventories (with the negative sign). Turning points calculated via the Bry-Boschan algorithm, based on the cyclical component of this indicator, were used as reference turning points in the first stage of the procedure. Also, a comparison of the results has been carried out between the HMM Signature Indicators and the RIED ICI.

The data sample covers the period from March 1997 to December 2021. There is a number of possible combinations of questions, which can lead to more or less adequate indicators. In addition, the size of the signature is another parameter necessary to be taken into account. From all combinations, 2 sets of questions were chosen. A different set of questions than that used in determining the RIED ICI was intentionally applied. Two indicators that will be described later in this section were calculated using the following input data and parameters:

- HMM IND 1 (Figure 2): size of the signature: 12, questions used: finished goods inventories (stock), selling prices of products (price);
- HMM IND 2 (Figure 3): size of the signature: 18, questions used: level of employment (employ), financial standing (fin), general economic situation in Poland (gen).

[2] See RIED (n.d.).

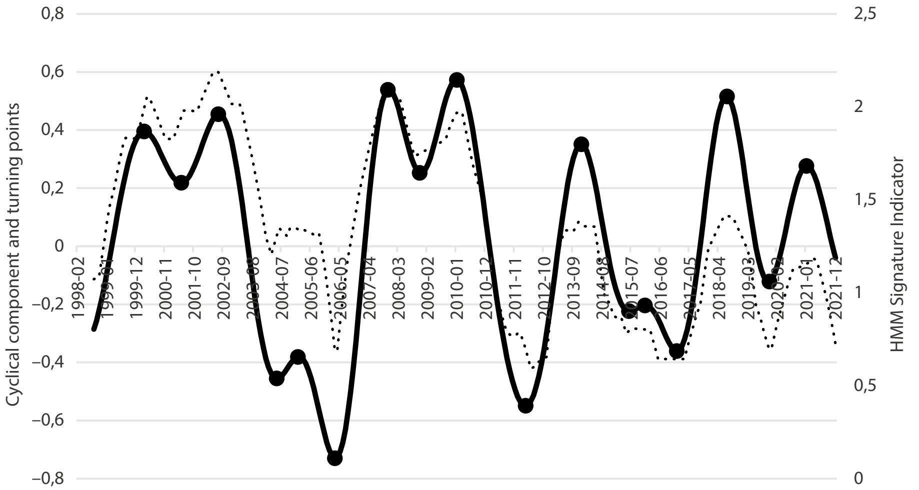

Figure 3. The HMM IND 2 (dotted line) along with the cyclical component (continuous line) and turning points (points) from March 1997 to December 2021
Source: Own calculation.

Two indicators differ from each other, both regarding size of the signature as well as the set of input time series. The results, however, are comparable (see Figure 4 and Table 1).

Based on the figures and the table, some conclusions may be drawn. First of all, despite the fact that both HMM Signature Indicators turned out to be quite smooth, a step with isolating a cyclic component is necessary. Without this step, each, even small change, would be considered as a turning point, which would lead to over-identification.

Secondly, all turning points detected by HMM IND 1 and HMM IND 2 have their counterparts in the RIED ICI, except the points at the beginning of the considered period, due to the shortening of the time series related to the size of the signature (the larger the size, the shorter the indicator). The number of turning points depends on the cyclical component which, in turn, is determined by the Christiano-Fitzgerald filter parameters (minimum and maximum period of oscillation).

The third remark concerns the time shifts between turning points identified on the basis of different indicators. The RIED ICI is recognised as a leading indicator. Meanwhile, both HMM Signature Indicators caught turning points much earlier than the RIED ICI (see Table 1). Only at the beginning of the considered period (HMM IND 1) or around periods where more turning points are identified according to the HMM IND 2, compared to the RIED ICI, were the turning points caught at a later time. It is, however, understandable, because just before the point was potentially caught too late, other turning points were located. Therefore, datings

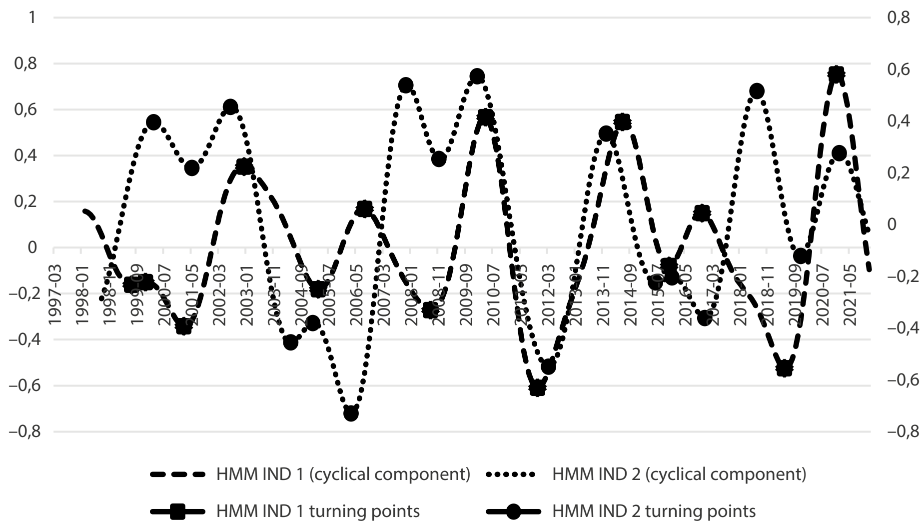

Figure 4. Comparison of the cyclical components regarding 2 HMM Signature Indicators with turning points identified according to the presented procedure. The HMM IND 2 (dotted line) along with the cyclical component (continuous line) and turning points (points) from March 1997 to December 2021

Source: Own calculation.

are still much before the actual shift in economic climate, and the overarching goal is fulfilled.

The fourth observation is related to the latest months in the analysed period. The RIED ICI at the end of 2021 still does not capture the peak and the last identified turning point is the trough in March 2020. Meanwhile, both HMM Signature Indicators have already registered another turning point (peak), just a month apart (HMM IND 1 in December 2020, and HMM IND 2 in January 2021). Thus, not only were all turning points caught much earlier, giving enough time for policymakers to take some action, but we also have an early warning signal of the impending deterioration of the situation.

The fifth conclusion is based on a comparison between 2 considered HMM Signature Indicators. The input time series was completely different, but the resulting sets of turning points were quite similar (see Figure 4). This suggests a certain redundancy of information contained in the responses to the survey questions. However, the graphs of differences between percentages of positive and negative answers to the questions from the RIED business tendency survey vary significantly, so instead of data redundancy, the cause should be sought in discovering non-obvious patterns in the raw data. The fact that there is no need to transform the input data is worth emphasizing.

Table 1. List of turning points in the Polish industry between the years 1997–2021, based on RIED ICI and HMM Signature Indicators (HMM IND 1 and HMM IND 2). For the corresponding turning points, the number of months between the dates is given. Negative values mean that the date of the turning point related to the corresponding HMM Signature Indicator is earlier than the date of the turning point related to RIED ICI

RIED ICI	Turning point	HMM IND 1	Shift [Mo.]	HMM IND 2	Shift [Mo.]
1998-12	Trough	1999-07	7	–	–
2000-03	Peak	1999-12	–3	2000-03	0
2001-08	Trough	2001-02	–6	2001-05	–3
2004-01	Peak	2002-12	–13	2002-07	–18
2005-06	Trough	2005-03	–3	2004-05	–13
–	Peak	–	–	2005-01	–
–	Trough	–	–	2006-03	–
2007-04	Peak	2006-08	–8	2007-11	7
2009-03	Trough	2008-08	–7	2008-11	–4
2010-11	Peak	2010-04	–7	2010-01	–10
2012-10	Trough	2011-11	–11	2012-03	–7
2014-08	Peak	2014-06	–2	2013-12	–8
2015-11	Trough	2015-11	0	2015-06	–5
–	Peak	–	–	2015-12	–
–	Trough	–	–	2016-12	–
2018-05	Peak	2016-11	–18	2018-07	2
2020-03	Trough	2019-05	–10	2019-11	–4
–	Peak	2020-12	–	2021-01	–

Source: Own calculations.

The proposed combinations of variables as input to the HMM Signature Indicator are probably not optimal, but chosen from thousands of possibilities. The difficulty is, nonetheless, found not in the procedure itself, but in the weakness related to the lack of hard data that can be used as a reliable reference series. Finding the right combination of variables, procedure parameters, but most of, all proper macro-economic data (instead of data from the business tendency survey), is a non-trivial task beyond the scope of this article, where empirical analysis was used as an illustration and validation of the proposed procedure.

Conclusions

The main objective of this paper was to present a modification of the original procedure described in (Bernardelli, 2020). With this modification, not only does the procedure not lose its effectiveness and versatility of application, but also, it becomes easier to define and more straightforward in usage. It applies the concept of the Christiano-Fitzgerald filter to obtain the cyclical component of the indicator, and afterwards, the Bry-Boschan algorithm to calculate the turning points. The effectiveness of the proposed procedure is illustrated by empirical analysis conducted based on the business tendency survey in the Polish manufacturing industry, carried out monthly by the RIED. As a reference, turning points identified, based on the RIED industrial confidence indicator, were used. In the empirical analysis, 2 HMM Signature Indicators were constructed with the use of different input time series. Despite this, both showed leading properties.

The proposed procedure of turning point identification, exploring the idea of signatures, proved to be a flexible and universal approach depending on only a few parameters:

- input time series and related number of variables included in the signature;
- size of the signature;
- size of the HMM state space;
- set of turning points needed for the construction of the reference signature;
- range of oscillation period in the Christiano-Fitzgerald filter.

By setting these parameters, we obtain, on the one hand, an indicator with great potential to be a leading one, and on the other, turning point identification, which with high probability, may be treated as an early warning signal.

Following the remarks from the original article (Bernardelli, 2020), the list of concluding observations can be further extended.

- The proposed approach to indicator construction is flexible due to easily parameterisable input, but because of the large number of possible combinations, finding the optimal parameters may be computationally complex.
- A direct consequence of the HMM's low requirements, compared to the classical econometric methods, is wide applicability of the procedure for turning point detection. Signatures exploit the anticipated leading properties of the HMM. Therefore, the constructed indicators have great potential to be early warning signs.
- The signature, the HMM Signature Indicator and related turning points are easily interpretable within the scope of the business cycle theory.
- The signature concept is suitable for any type of data, both macro-economic and that derived from surveys. It can be also mixed in 1 signature. The presented solution is also resistant to incompatible data, e.g. shorter time series can be used with longer ones at the same time.

There are also many possibilities for generalisation or modification at almost every stage of the described procedure. Some examples are listed below:

- at the signature construction stage, instead of the classical arithmetic mean, the weighted mean can be implemented;
- there could be separate signatures for peaks and troughs. One reference signature would then be associated with the periods of relatively good conditions, while the other; with a worse situation;
- the RMSE measure of distance between the signatures may be replaced by a variety of measures;
- at the turning point identification stage, the asymmetric Christiano-Fitzgerald filter can be replaced by an alternative method of time series decomposition, such as: STL, Tramo-Seats or X-13-ARIMA-SEATS.

The next, natural step in research in this area will be verification of accuracy regarding indicators and turning points calculated following the procedure proposed in this article.

Appendix

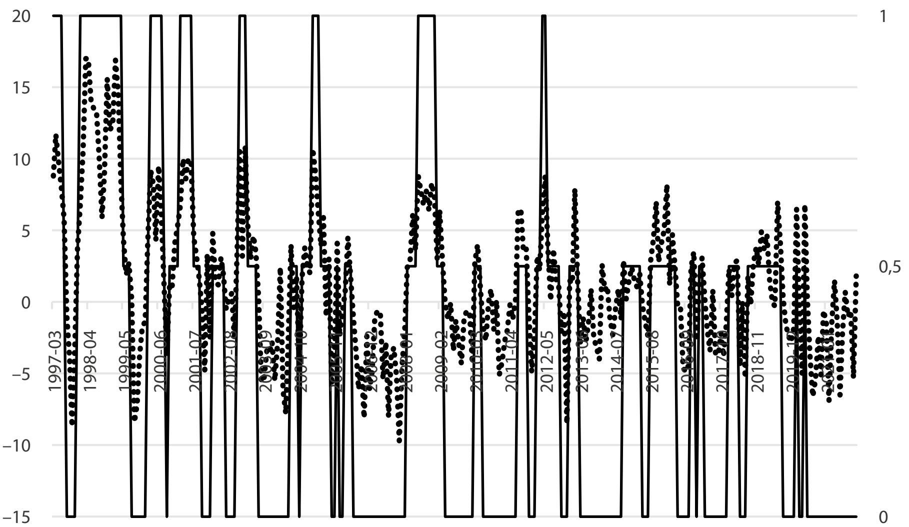

Figure 5. The difference between percentages of positive and negative answers to the question from the RIED business tendency survey about finished goods inventories in the Polish manufacturing industry (dotted line) along three the three-state VP (continuous line) from March 1997 to December 2021

Source: Own calculation based on RIED data.

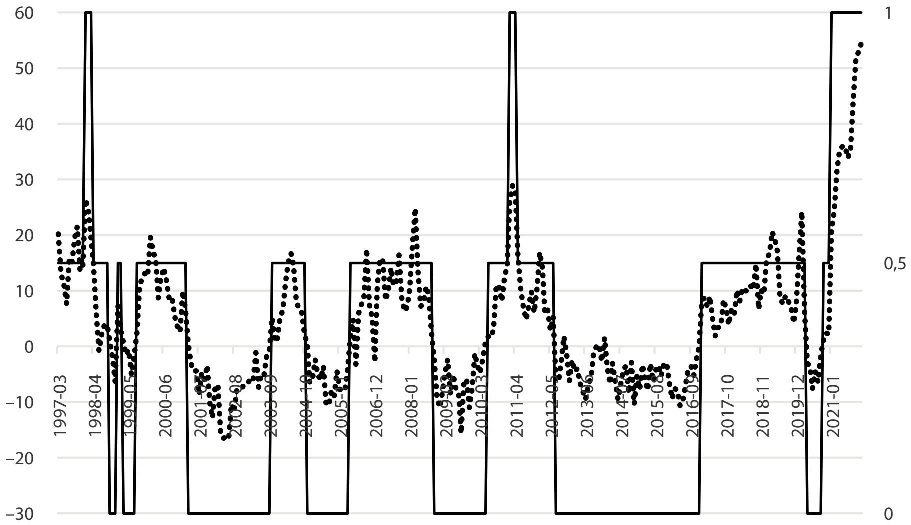

Figure 6. The difference between percentages of positive and negative answers to the question from the RIED business tendency survey about selling prices of products in the Polish manufacturing industry (dotted line) along with the 3-state VP (continuous line) from March 1997 to December 2021

Source: Own calculation based on RIED data.

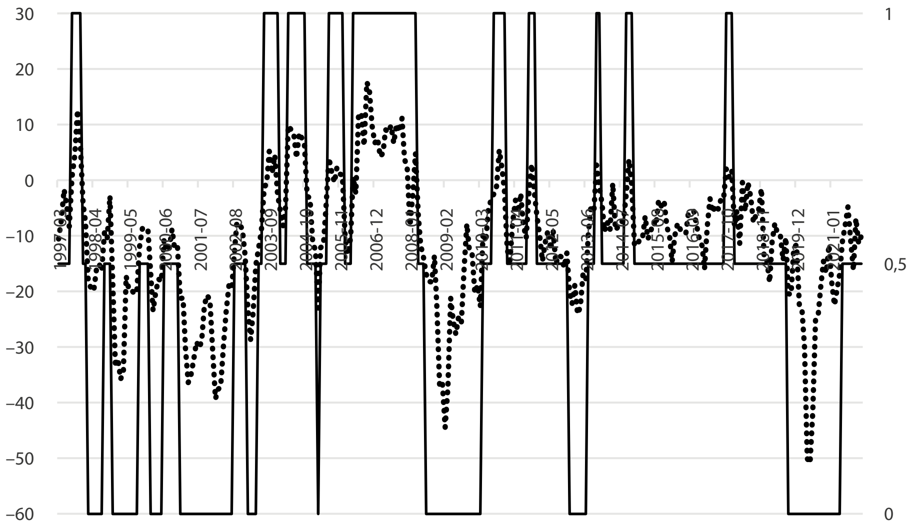

Figure 7. The difference between percentages of positive and negative answers to the question from the RIED business tendency survey about financial standing in the Polish manufacturing industry (dotted line) along with the 3-state VP (continuous line) from March 1997 to December 2021

Source: Own calculation based on RIED data.

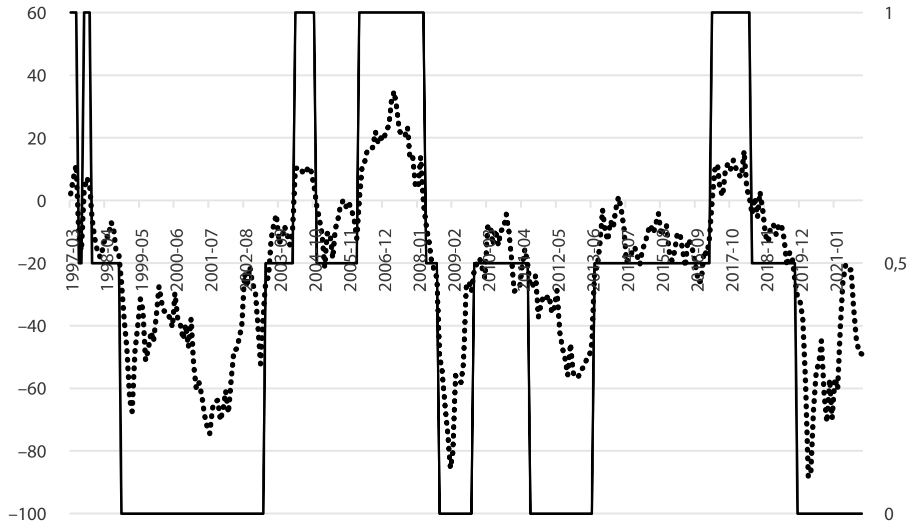

Figure 8. The difference between percentages of positive and negative answers to the question from the RIED business tendency survey about general economic situation in Poland (dotted line) along with the 3-state VP (continuous line) from March 1997 to December 2021

Source: Own calculation based on RIED data.

References

Abberger, K., & Nierhaus, W. (2010). Markov-switching and the Ifo business climate. The Ifo business cycle traffic lights. *Journal of Business Cycle Measurement and Analysis*, *1*(2), 1–13.

Adamowicz, E., Dudek, S., Kluza, S., Ratuszny, E., & Walczyk, K. (2019). Koniunktura gospodarcza i bankowa w Europie Środkowo-Wschodniej. In M. Strojny (Ed.), *Europa Środkowo-Wschodnia wobec globalnych trendów: gospodarka, społeczeństwo i biznes* (pp. 353–383). Warszawa: Oficyna Wydawnicza SGH.

Baum, L. E., Petrie, T., Soules, G., & Weiss, N. (1970). A maximization technique occurring in the statistical analysis of probabilistic functions of Markov chains. *The Annals of Mathematical Statistics*, *41*(1), 164–171.

Bernardelli, M. (2015). The procedure of business cycle turning points identification based on hidden Markov models. In K. Walczyk (Ed.), *Analyzing and forecasting economic fluctuations* (pp. 5–23). Warszawa: IRG SGH.

Bernardelli, M. (2020). Economic climate HMM-based indicator as a tool for the business cycle turning points signalling. In M. Męczarski (Ed.), *Metody ekonometryczne, statystyczne i matematyczne w modelowaniu zjawisk społecznych*, vol. 1: *Metody probabilistyczne w zastosowaniach ekonomicznych* (pp. 189–208). Warszawa: Oficyna Wydawnicza SGH.

Bernardelli, M., Próchniak, M., & Witkowski, B. (2021). *Economic growth and convergence: Global analysis through econometric and hidden Markov models*. https://doi.org/10.4324/9781003171645

Boldin, M. D. (1994). Dating turning points in the business cycle. *Journal of Business*, *67*(1), 97–130.

Bry, G., & Boschan, C. (1971). *Programmed selection of cyclical turning points*. (NBER Working Paper No. 1971, Cyclical analysis of time series: Selected procedures and computer programs, pp. 7–63).

Cappé, O., Moulines, E., & Rydén, T. (2005). *Inference in hidden Markov models*. New York: Springer-Verlag.

Chauvet, M., & Hamilton, J. D. (2005). *Dating business cycle turning points.* (NBER Working Paper No. 11422).

Christiano, L., & Fitzgerald, T. J. (2003). The bandpass filter. *International Economic Review*, *44*(2), 435–465.

Cleveland, R. B., Cleveland, W. S., McRae, J. E., & Terpenning, I. J. (1990). STL: A seasonal-trend decomposition procedure based on loess. *Journal of Official Statistics*, *6*(1), 3–73. Retrieved from http://bit.ly/stl1990

Dagum, E.B., & Bianconcini, S. (2016). *Seasonal adjustment methods and real time trend-cycle estimation*. Springer.

Dufrénot, G., & Keddad, B. (2014). Business cycle synchronization in East Asia: A Markov switching approach. *Economic Modelling*, *42*, 186–197.

Federal Reserve Bank of St. Louis. (n.d.). *OECD based recession indicators for Poland from the peak through the trough (POLRECDM)*. Retrieved January 24, 2022 from https://fred.stlouisfed.org/series/POLRECDM

Hamilton, J. D. (1994). *Time series analysis*. Princeton: Princeton University Press.

Milas, C., Rothman, P. A., Dijk, D., & Wildasin, D. E. (2006). *Nonlinear time series analysis of business cycles.* Emerald Group Publishing.

Nilsson, R., & Gyomai, G. (2011). *Cycle extraction*. Retrieved January 24, 2022 from http://www.oecd.org/dataoecd/32/13/41520591.pdf

OECD. (2022). *Composite leading indicator (CLI)*. https://doi.org/10.1787/4a174487-en

RIED. (n.d.). *Metody badawcze. kwestionariusze ankiet*. Retrieved January 24, 2022 from http://kolegia.sgh.waw.pl/pl/KAE/struktura/IRG/koniunktura/Strony/metody.aspx

Sax, C., & Eddelbuettel, D. (2018). Seasonal adjustment by X-13ARIMA-SEATS in R. *Journal of Statistical Software*, *87*(11), 1–17. https://doi.org/10.18637/jss.v087.i11

Smith, P. A., & Summers, P. M. (2005). How well do Markov switching models describe actual business cycles? The case of synchronization. *Journal of Applied Econometrics*, *20*(2), 253-274.

Viterbi, A. (1967). Error bounds for convolutional codes and an asymptotically optimum decoding algorithm. *IEEE Transactions on Information Theory*, *13*, 260–269.